Numerical Solution for Blowdown of Pipeline Containing Flashing Liquid

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An intentional or accidental opening in a pipeline containing a subcooled liquid at a high pressure may produce its rapid depressurization (blowdown). The liquid starts to flash when its pressure reaches the local saturation pressure and flows thereafter as a two-phase mixture. The modeling of this process plays a crucial role in the analysis of accident scenarios and in the design of controlled depressurization systems in chemical, nuclear, and offshore industries.

Prediction of transfer of heat from the pipe wall to the two-phase flow during the depressurization process is a transient conjugated heat-transfer problem since the heat-transfer rate at the wall-fluid interface and local fluid conditions are directly dependent on each other and need to be simultaneously calculated. A number of approaches to the solution of this problem have been developed for a single-phase pipe flow. Approximation methods in which the thermal capacity of the pipe wall is assumed negligible were proposed by Siegel (1960), Sucec (1975), and Li (1986). Based on the lumped capacitance approach to the transient heat conduction in the wall, Sucec (1981) and Lin (1991) obtained exact analytical and numerical solutions for a finite wall thermal capacity. Later, Lin and Kuo (1988) included the effect of the internal thermal resistance of the wall by using a one-dimensional (1-D) model of transient heat conduction in the radial direction.

Much of the past work on the pipeline blowdown has concentrated on the development of models describing the hydrodynamics of the two-phase pipe flow. In these models the flow is assumed either adiabatic or isothermal (Grolmes et al., 1984; Richardson and Saville, 1991). Chen et al. (1995) formulated a model for a two-phase blowdown which takes into account forced convection heat transfer between the pipe wall and the fluid flow. However, the effects of wall thermal capacity and transient conduction are assumed negligible in their study. This approach can be used for heat transfer in flows bounded by very thin walls. The ratio of wall thickness to the internal diameter for steel pipes employed in the transport of flashing liquids reaches 0.13 (Engineering Data Book, 1972) and, therefore, a large amount of heat may be transferred to the fluid during a pipeline blowdown. Solving the conjugate heat-transfer problem for a flashing flow in a pipe is a complex problem due to difficulties associated with the modeling of forced convection in the conditions of two-phase flow. The traditional approach to the solution of this problem is based on the use of empirical two-phase forced convection correlations in conjunction with a 1-D or 2-D numerical model of heat conduction in the transverse direction (Erickson and Mai, 1992). Recently, Fairuzov (1998) developed a novel approach to the description of heat transfer in long two-phase pipelines in which the effect of the thermal capacitance of the pipe wall ($\rho_w c_w V_w$) is incorporated into the energy equation for the fluid flow in the form of an additional term.

In this note, based on the approach developed in a previous article (Fairuzov, 1998), a numerical solution for a blowdown of a long pipeline conveying a flashing liquid is found using a finite-difference method. The obtained solution is verified through comparison with previously published experimental data for a two-phase blowdown from a 100 m pipeline containing a two-component mixture. A dimensionless number for evaluating the effects of the conjugated heat transfer on the fluid flow is introduced. Numerical simulations demonstrate the effect of the wall thermal capacitance on the flashing flow behavior in pipes with an extremely high length-to-diameter ratio.

Mathematical Formulation

The system to be examined is a horizontal pipeline with internal diameter D, length L, and the thickness of the pipe wall δ, as shown in Figure 1. The pipeline is filled with a

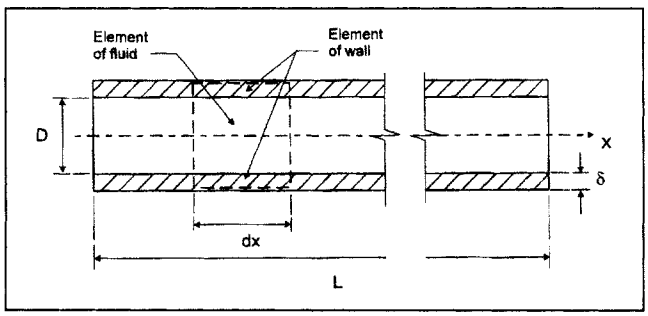

Figure 1. Pipeline geometry and control volume with elements of fluid and pipe wall.

subcooled liquid at a high pressure ($p > p_a$). Initially, the fluid and the pipe walls are at a uniform temperature, and both ends of the pipeline are closed. At $t = 0$, a full-bore rupture at one of the ends of the pipeline initiates the depressurization process. The other end of the pipeline remains closed. During a very short period of time, a number of complex physical processes occur in the single-phase liquid flow (depressurization wave propagation, (Reibold et al., 1981); pressure undershoot, (Alamgir and Leinhard, 1981)). For long pipelines, the modeling of these processes is usually of no real practical significance; therefore, these are not considered in this study. After passage of the depressurization wave, the fluid pressure is very close to the saturation pressure corresponding to the initial fluid temperature. The liquid starts to flash at the ruptured end of the pipeline. The flashing front propagates from the ruptured end to the closed end of the pipeline.

To facilitate the analysis, the following simplifying assumptions are made:

(1) The flow is one-dimensional.

(2) The homogeneous equilibrium model can be used for description of the two-phase flow in the pipeline. This was demonstrated by a number of researchers (Wallis, 1981; Chen, 1995).

(3) The fluid is in local thermal equilibrium with the pipe wall. This assumption is valid for long pipelines ($L/D > 10^4$) (Fairuzov, 1998).

(4) In the pipe wall, the heat transfer by conduction in the axial direction is negligible. The temperature gradients in axial direction are negligible in comparison with the radial gradients.

With these assumptions, the governing equations for a control volume comprising an infinitesimal element of the fluid and an element of the wall (see Figure 1) can be written as follows

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V) = 0 \tag{1}$$

Momentum equation

$$\rho \frac{\partial V}{\partial t} + \rho V \frac{\partial V}{\partial x} + \frac{\partial p}{\partial x} + F_w = 0 \tag{2}$$

Energy equation

$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{V_{cv}} \frac{\partial}{\partial t}(M_w u_w) + \frac{\partial}{\partial x}(\rho u V) + p \frac{\partial V}{\partial x} = q \tag{3}$$

where V_{cv} is the volume of the fluid element (Adx). The second term on the left side of Eq. 3 is the rate of change of the internal energy of the element of the wall per unit volume of the fluid. This term takes into account the transfer of heat from the pipe wall to the fluid flow during pipeline blowdown.

Since the internal energy of the pipe wall depends only on its temperature, Eq. 3 may be rewritten as follows

$$\frac{\partial(\rho u)}{\partial t} + \frac{M_w c_w}{V_{cv}} \frac{\partial T}{\partial t} + \frac{\partial}{\partial x}(\rho u V) + p \frac{\partial V}{\partial x} = q \tag{4}$$

If the fluid is a multicomponent mixture, its temperature is a function of p and u or ρu. Therefore, Eq. 4 can be rearranged in the following form

$$\frac{\partial(\rho u)}{\partial t} + \frac{M_w c_w}{V_{cv}} \left\{ \left(\frac{\partial T}{\partial p} \right)_{\rho u} \frac{\partial p}{\partial t} + \left(\frac{\partial T}{\partial (\rho u)} \right)_p \frac{\partial (\rho u)}{\partial t} \right\} + \frac{\partial}{\partial x}(\rho u V) + p \frac{\partial V}{\partial x} = q \tag{5}$$

The temperature of a single component two-phase mixture depends only on its pressure; thus Eq. 5 may be reduced to

$$\frac{\partial(\rho u)}{\partial t} + Fa \frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho u V) + p \frac{\partial V}{\partial x} = q \tag{6}$$

where

$$Fa = \rho_w c_w \left(\frac{dT}{dp} \right)_{\text{sat}} \frac{A_w}{A} \tag{7}$$

Examination of Eq. 7 reveals that the effect of heat transfer from the wall to the fluid is determined by the geometry of the pipeline, thermodynamic properties of steel, and the rate of change of saturation temperature with respect to saturation pressure of the liquid. Fa is a fundamental dimensionless number governing the effect of thermal capacity of the pipe wall on the behavior of flashing liquid flow. It is similar to the wall-to-fluid thermal capacity ratio that is commonly used in the analysis of conjugated heat transfer in a single-phase flow.

Substituting the Clausius-Clapeyron equation in Eq. 7 and expanding the last term on its right side results in

$$Fa = \frac{4 \rho_w c_w T (v_G - v_L)}{h_{fg}} \left[\frac{\delta}{D} + \left(\frac{\delta}{D} \right)^2 \right] \tag{8}$$

As can be seen from Eq. 8, Fa is a ratio of two effects: pipe-wall thermal capacity/latent heat of evaporation.

For a multicomponent two-phase flow, it is more convenient to have the energy equation in terms of specific enthalpy rather than specific internal energy since a pressure-enthalpy flash calculation is usually performed to evaluate the thermodynamic properties of the fluid. Thus, Eq. 5 can be rewritten as follows

$$(1 + Fa_2) \frac{\partial(\rho h)}{\partial t} + (Fa_1 - 1) \frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho h V) - V \frac{\partial p}{\partial x} = q \tag{9}$$

where

$$Fa_1 = 4 \rho_w c_w \left(\frac{\partial T}{\partial p} \right)_{ph} \left[\frac{\delta}{D} + \left(\frac{\delta}{D} \right)^2 \right] \tag{10}$$

$$Fa_2 = 4 \rho_w c_w \left(\frac{\partial T}{\partial (\rho h)} \right)_p \left[\frac{\delta}{D} + \left(\frac{\delta}{D} \right)^2 \right] \tag{11}$$

In this case, two dimensionless parameters, Fa_1 and Fa_2, govern the amount of heat transferred to the fluid flow. The examination of Eq. 9 reveals that the multicomponent flashing liquid flow can be assumed adiabatic $Fa_1 \ll 1$ and $Fa_2 \ll 1$. It should be noted that $Fa_2 \ll Fa_1$ for most multicomponent mixtures since $(\partial T/\partial p)_{\rho h} \gg (\partial T/\partial(\rho h)/$. Thus, only Fa_1 is used for evaluating the thermal effect of the wall and the subscript 1 is omitted in the subsequent sections of this article.

The equation of state can be written as

$$\rho = \rho(p, h, z) \tag{12}$$

The thermodynamic and transport properties of the fluid are calculated using a computer program for predicting multicomponent vapor–liquid equilibria developed by Solorzano et al. (1996).

The drag force per unit volume of the fluid which is exerted by pipe wall on the two-phase mixture can be expressed as follows

$$F_w = /\Phi_{LO}^2 \frac{f_{LO}}{2D} \frac{G^2}{\rho_L} \tag{13}$$

where the two-phase multiplier is evaluated using the correlation of McAdams (1942).

The boundary conditions are (for $t > 0$)

at the intact end, $x = 0$: $G = 0$;
at the ruptured end,

$$x = L \quad \begin{cases} \text{choked flow } (p > p_a): G = G(p, h, z) \\ \text{subsonic flow: } p = p_a \end{cases}$$

The choked flow at the rupture is predicted using the model of Starkman et al. (1964) extended to multicomponent mixtures.

The initial conditions are at $t = 0$: (for $0 < x < L$)

$$V = 0,\ p = p_i,\ T = T_a$$

Numerical Method

The system of conservation equations Eq. 1, 2, 5, or 9 is solved using the RELAP5 numerical solution scheme (Ransom and Trapp, 1978). The method is based on replacing the system of differential equations with a system of finite-difference equations, which are partially implicit in time. The scalar properties (pressure, density, internal energy or enthalpy) are defined at the cell centers, and vector quantities are defined at the cell junctions. All implicit terms are linear in the new time variables. To ensure stability of the numerical scheme, a donor formulation is used to compute mass and energy fluxes. A donor-like formulation is used to define the momentum flux term. The three field equations are reduced to a single difference equation per mesh cell in terms of pressure. The resultant system of N algebraic equations is solved using the Thomas algorithm. All calculations were carried out on a Sun Ultra 1 workstation.

Results and Discussions

To verify the present numerical solution, the experimental data reported by Tam and Cowley (1988) were used. They conducted a series of experiments on pressurized liquefied petroleum gas (LPG) releases using two 100-m-long pipelines of internal diameter 50 mm and 150 mm. A detailed description of these experiments can be found in the works of Richardson and Saville (1996) and Chen et al. (1995). The measurements for a full-bore rupture of the test pipeline of internal diameter 150 mm were taken to validate the model. The pipeline was filled with the LPG. Tam and Cowley (1988) do not report the exact composition of LPG used in their experiment. Therefore, the LPG has been assumed to comprise 95 mol. % propane and 5% butane. The initial pressure and temperature were 11.25 bar and 19.9°C, respectively. The assumed roughness length-scale is 0.05 mm and the assumed material of the pipe wall is 1% carbon steel. Since the L/D ratio of the test pipeline was relatively small (667), the total blowdown time was too short (25 s) for the thermal equilibrium within the pipe wall to be achieved. Therefore, a rigorous conjugated heat-transfer model should be used for modeling of heat transfer in the test pipeline. Nevertheless, the proposed approach can also be used to take the thermal effect of the pipe wall into account approximately. For this purpose, it was assumed that only a part of the pipe wall adjacent to the wall-fluid interface is in the local thermal equilibrium with the fluid. Estimations made in the previous article (Fairuzov, 1998) show that the thickness of this thermally penetrated layer, averaged over the total blowdown time and the pipe length, is 1.9 mm. This value of δ corresponds to $Fa = 7.6$. A comparison of the numerical results for the mass of the fluid in the pipeline (inventory), pressure at the closed end, and pressure at the open end of the line with experimental measurements is shown in Figures 2 and 3. As can be observed, the present numerical calculations agree fairly well with the experimental measurements. The model does not predict the small pressure undershoot in the begin-

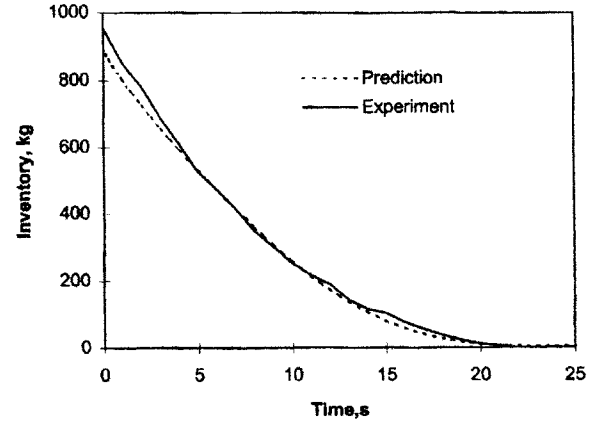

Figure 2. Predicted vs. measured variations of inventory of line with time.

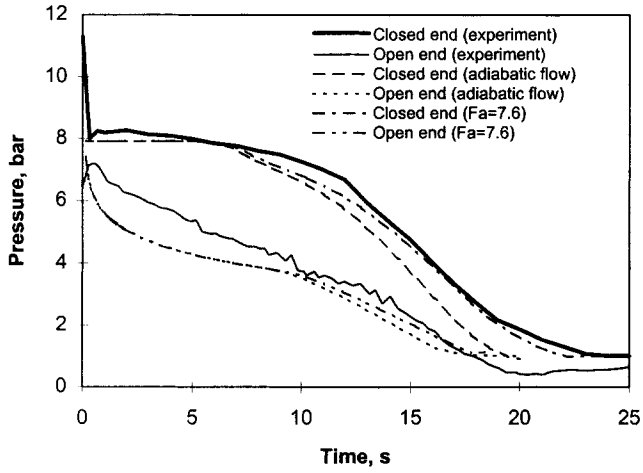

Figure 3. Predicted vs. measured variations of pressure at the closed and open ends of the line.

ning of the blowdown process, because the dynamics of the depressurization wave is not considered in this study. It is seen from Figure 3 that the adiabatic flow model ($Fa = 0$) significantly overpredicts the rate of pressure drop at the closed end of the test line.

To assess effects of the pipe wall on the fluid flow during a long pipeline blowdown, numerical simulations of a full-bore rupture of a horizontal pipeline ($L/D = 7.9 \cdot 10^4$) containing pure propane were carried out. The results of the calculations are presented in terms of dimensionless temperature drop θ/θ_{max} and Fa in Figure 4. The temperature drop was calculated at the end of the depressurization process, that is, when 99% of the initial mass of the fluid had been released from the pipeline. As can be seen, the drop in temperature at the ruptured end of the pipeline does not depend on Fa. It can be explained by the fact that the pressure at the ruptured end is equal to the ambient pressure since the flow is always subsonic by the end of the blowdown. Therefore, the fluid temperature at the rupture is equal to the saturated temperature corresponding to the ambient pressure and does not depend on the heat-transfer conditions in the pipeline. The drop

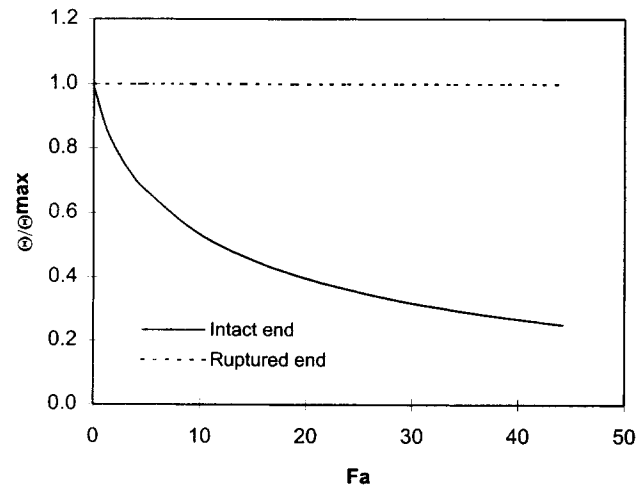

Figure 4. Effect of *Fa* on drop in fluid temperature.

in fluid temperature at the intact end of the pipeline strongly depends on Fa. The thermal effect of the wall can be neglected when $Fa \ll 1$. This fact confirms the theoretical results presented in the second section of this article.

Conclusions

A numerical solution for a blowdown of a long pipeline carrying a flashing liquid has been obtained. A fundamental dimensionless number for evaluating the effects of conjugated heat transfer on the fluid flow has been introduced. The predictions based on the obtained solution have been compared to previously published experimental data for a blowdown of a pipeline containing pressurized LPG. The present predictions agree well with the experimental measurements. The significance of the effect of the thermal capacitance of the pipe wall has been demonstrated. The technique developed can be readily applied to study related transient two-phase flow problems.

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Notation

A = area
c = specific heat
f = friction factor
F_w = friction force per unit volume of the fluid
G = mass velocity
h = specific enthalpy
h_{fg} = latent heat of vaporization
M = mass
N = total number of mesh cells
q = external heat flux per unit volume of the fluid
T = temperature
u = specific internal energy
v = specific volume
z = vector of overall mixture composition
θ = drop in the fluid temperature $(T_i - T)$
θ_{max} = maximum drop in the fluid temperature $[T_i - T_{sat}\ (P_a)]$

Subscripts

a = ambient
i = initial conditions
L = liquid
G = vapor (gas)
LO = liquid only
sat = saturation
w = pipe wall

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Corrections

• In the article titled "Detection of Gross Errors in Data Reconciliation by Principal Component Analysis" (July 1995, p. 1712), the expression on line 4 up from the bottom of p. 1715 that read $c_\alpha h_0 < 0$ is incorrect and should read $c_\alpha h_0 > 0$. This error does not affect the rest of the article.

• In the article titled "Lumen Mass Transfer in Hollow-Fiber Membrane Processes with Nonlinear Boundary Conditions" (April 1998, p. 836), Eq. 24 should read

$$2\pi RN = 2\pi r D_0 (1 + D_1 C_{A,m}) \frac{dC_{A,m}}{dr}, \quad (R \le r \le R_1) \qquad (24)$$

• In the article titled "Hydrodynamic Model for Gas-Lift Reactors" (June 1998, p. 1413), Eqs. 16 and 22 should read

$$f_r = \frac{0.187\sqrt{\bar{\epsilon}}}{(1-\bar{\epsilon})^{0.10}} \left(\frac{\sqrt{gD}}{\bar{V}_l} \right)^{1.1} \qquad (16)$$

$$\left\{ \left[\left(\frac{D_{dc}}{D} \right)^4 - 1 \right] + f_{dc} \frac{L_{dc}}{D_{dc}} + \sum_i K_i \right\} \frac{\bar{v}_{cc}^2}{2g} + \frac{\langle \bar{P} \rangle_{z=0} - \bar{P}_{dc,z=L}}{p_l g} - L = 0 \qquad (22)$$

↓ kinetic energy change (downcomer to riser) ↓ wall friction ↓ accessories ↓ pressure contribution ↓ gravitational contribution